Proceedings of the 1998 IEEE
International Conference on Control Applications
Trieste, Italy 1-4 September 1998

Constraint on Steady State Output Imposed by Zeros at $s = 0$ and Servo Synthesis Using Unstable Weight with Application to Multi Area Frequency Control of Power Plants

Sergio B. Villas-Boas and Kang-Zhi Liu
Department of Electronics and Mechanical Engineering
Chiba University
1-33 Yayoi-cho, Inage-ku, Chiba 263-8522
Fax +81-43-290-3039 Email liu@cute.te.chiba-u.ac.jp

Abstract

When an MIMO plant has zeros at $s = 0$, it will be shown that this imposes a constraint on the steady state output of the plant. Therefore, reference tracking/disturbance rejection is possible only if certain structural condition is satisfied. In the optimal servo syntheses using unstable weight, such as the one using Unstably Weighted $\mathcal{H}_\infty$ Control, the structure of unstable weight can not be selected freely in this case. The required structure is exposed in this paper.

Further, the proposed method is applied to the multi area frequency control of power plants which has a zero at $s = 0$.

Keywords: Plant having zero at $s = 0$, steady state constraint, structural constraint on unstable weight, multi area frequency control, power plant, unstably weighted $\mathcal{H}_\infty$ Control.

1 Introduction

When an MIMO plant has zeros at $s = 0$, it is well known that this imposes a constraint on the structure of unstable dynamics of the controller. In this paper, we revisit this issue from a new standpoint, i.e. to expose the constraint on the steady state output of the plant due to this unstable zero. This, in reference tracking/disturbance rejection problems, implies that the reference/disturbance is not free and is subject to a structural constraint. This study is necessary in the servo design using unstable weight. It will be shown that the unstable weight must also satisfy a structural constraint in this case.

Then Ustably Weighted $\mathcal{H}_\infty$ Control[6, 5] can be applied to design a controller for asymptotic rejection of step disturbance or reference tracking.

On the other hand, the frequency control problem of multi area power plants is an important age-old problem in power industry. The main issue of this problem is to attenuate the frequency deviations due to the change of load (modeled as step disturbance). The control system must be robust to parameter uncertainties as well. This problem has been tackled by using a variety of approaches[1, 3, 7].

The difficulty associated with this problem is that the linearized model of a multi area electrical power plant having frequency deviations as outputs is an MIMO system that has a transmission zero at $s = 0$. In this paper, we apply the proposed method to this problem and design an $\mathcal{H}_\infty$ controller such that the steady state frequency deviations due to step load disturbances are controlled to zero. Some comparisons with other approaches are also made.

The notations used are standard. G_{yx} denotes the transfer matrix $x \mapsto y$.

2 Plant zero and steady state output constraint

Let $P(s)$ be an nth order plant of dimension $m \times p$ and let

$$P(s) = [A_p,\ B_p,\ C_p,\ 0] \tag{1}$$

be a stabilizable and detectable realization. Suppose the plant has n_z zeros at $s = 0$, $m > n_z \geq 1$. Then there exists a full row rank matrix $V = \begin{bmatrix} V_1 & V_2 \end{bmatrix}$, $\text{rank}(V) = n_z$, that satisfies the equation below

$$\begin{bmatrix} V_1 & V_2 \end{bmatrix} \begin{bmatrix} A_p & B_p \\ C_p & 0 \end{bmatrix} = 0 \tag{2}$$

$$Im\, C_p \cap Ker\, V_2 = \emptyset.$$

Due to the stabilizability of (A_p, B_p), V_2 must have full row rank.

It is also assumed that this plant is subject to a disturbance $d(t)$ in the following way:

$$\Sigma: \begin{cases} \dot{x}_p(t) = A_p x_p(t) + B_p u(t) + B_d d(t) \\ y(t) = C_p x_p(t) + D_d d(t). \end{cases} \quad (3)$$

Note if we take $d(t) = r(t)$, $D_d = -I$ and $B_d = 0$, then $y(t) = -(r - y_p)$ and the tracking problem can treated as a disturbance rejection problem.

Assumption 1 *The disturbance $d(t)$ is a vector of step signals with arbitrary direction.*

Theorem 1 *For the system Σ given above, the following statements hold.*
(i) the steady state output $y(\infty)$ must satisfy the constraint

$$V_2 \cdot y(\infty) = [V_1 \ V_2] \begin{bmatrix} B_d \\ D_d \end{bmatrix} d \quad (4)$$

when Σ is stabilized by feedback control.
(ii) The step disturbance $d(t)$ can be rejected asymptotically only if $Im \begin{bmatrix} B_d \\ D_d \end{bmatrix} \in Ker\ [V_1 \ V_2]$.

(Proof) (i) Owing to Assumption 1, it can be proved that $\dot{x}_p(\infty) = 0$ holds in steady state so that

$$\begin{aligned} 0 &= A_p x_p(\infty) + B_p u(\infty) + B_d d \\ y(\infty) &= C_p x_p(\infty) + D_d d. \end{aligned}$$

Then by (2) we have

$$[V_1 \ V_2] \begin{bmatrix} 0 \\ y(\infty) \end{bmatrix} = [V_1 \ V_2] \begin{bmatrix} B_d \\ D_d \end{bmatrix} d.$$

(ii) This follows from (i) since $y(\infty) = 0$. □

3 Servo synthesis using unstable weight

Let us consider the asymptotic rejection of step disturbance of system Σ. A useful way of doing this in the framework of optimal control is to use unstable weight at output y, as W_S shown in Fig. 1. This unstable weight represents the dynamics of the disturbance d.

W_S of (5) is the general form for a transfer matrix having integrator dynamics only. T_W is a design parameter and is to be determined. If fine tuning is required, extra stable weight can be added behind W_S or parallel to W_S. The results of this paper are not affected.

$$W_S = \frac{1}{s} \times T_W = \left[\begin{array}{c|c} 0_{n_1} & T_W \\ \hline I_{n_1} & 0 \end{array} \right]. \quad (5)$$

Assumption 2 *T_W has full rank.*

Note at this stage, it is not clear whether T_W should have full row rank or full column rank and how many rows T_W should have. This is to be determined so that a control system that rejects disturbance asymptotically can be synthesized by using the unstable weight.

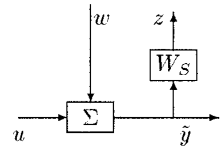

Figure 1: Generalized plant structure

3.1 Determination of T_W

The key idea of using unstable weight in the design is to ensure asymptotic disturbance rejection by comprehensive stability [4]. That is, to guarantee $y(\infty) = 0$ by the stability of unstably weighted closed loop transfer matrix G_{zw} (see Fig. 1). The following theorem gives the requirement on the unstable weight $W_S(s)$ in order to realize this purpose.

Theorem 2 *Assume $Im \begin{bmatrix} B_d \\ D_d \end{bmatrix} \in Ker\,[V_1 \ V_2]$. Let T_W be an $n_1 \times m$ constant matrix and suppose there exists a controller $K(s)$ stabilizing Σ internally. Further, let $y(t)$ be the response of Σ to the step disturbance d. Then $G_{zw}(s) \in \mathcal{RH}_\infty \Rightarrow y(\infty) = 0$ iff*

$$V_y = \begin{bmatrix} V_2 \\ T_W \end{bmatrix} \quad (6)$$

is nonsingular.

(Proof) Only if : First of all, we investigate the condition for $G_{zw}(s) \in \mathcal{RH}_\infty$. As the closed loop system (Σ, K) is internally stable, the generalized plant of Σ with respect to exogenious input w and controlled output z (Fig. 1) must be comprehensively stabilizable [4]. One of the comprehensive stabilizability condition requires that the unstable poles of W_S, which are not observable from $\tilde{y}$, be controllable from u. The transfer matrix $u \mapsto z$ has a realization as below

$$W_S P(s) = \left[\begin{array}{cc|c} A_p & 0 & B_p \\ T_W C_p & 0_{n_1} & 0 \\ \hline 0 & I_{n_1} & 0 \end{array} \right]. \quad (7)$$

We need only to find the controllability condition of this realization at $s = 0$. This requires

$$\text{rank } J(0) = n + n_1 \quad (8)$$

where

$$J(0) := \begin{bmatrix} A_p & 0 & B_p \\ T_W C_p & 0 & 0 \end{bmatrix}. \quad (9)$$

Suppose $\exists\ \eta = [\eta_1 \ \eta_2] \neq 0$ such that $\eta\ V_y = 0$, i.e.

$$\eta_1 V_2 = -\eta_2 T_W.$$

Obviously $\eta_2 \neq 0$ since otherwise $\eta_1 = 0$ would follow from $\eta_1 V_2 = 0$ and the fact that V_2 is row full rank, which contradicts the assumption $\eta \neq 0$.

So we have

$$\begin{aligned} &\left[\begin{array}{cc} \eta_1 V_1 & -\eta_2 \end{array}\right] J(0) \\ &= \left[\begin{array}{cc} \eta_1 V_1 & \eta_1 V_2 \end{array}\right] \left[\begin{array}{ccc} A_P & 0 & B_P \\ C_p & 0 & 0 \end{array}\right] \\ &= 0. \end{aligned} \quad (10)$$

This means that if V_y does not have full row rank, at least a pole of W_S at $s = 0$ is not controllable. Therefore, $G_{zw} \in \mathcal{RH}_\infty$ is possible only if V_y has full row rank.

Now, suppose $G_{zw} \in \mathcal{RH}_\infty$. Since $G_{yd} = G_{\tilde{y}w}$ and

$$G_{zw}(s) = \frac{1}{s} T_W G_{\tilde{y}w} = T_W G_{yd} \times \frac{1}{s} I,$$

the response $y(t)$ to any step disturbance $d(t)$ satisfies

$$T_W \cdot y(\infty) = 0 \quad (11)$$

in steady state. According to Theorem 1, under the given assumption on the disturbance matrix we have $V_2\ y(\infty) = 0$. So,

$$\left[\begin{array}{c} V_2 \\ T_W \end{array}\right] \cdot y(\infty) = 0 \quad (12)$$

holds. Then, it is obvious that $G_{zw} \in \mathcal{RH}_\infty \Rightarrow y(\infty) = 0$ only if V_y has full column rank.

Therefore, we conclude that $G_{zw} \in \mathcal{RH}_\infty \Rightarrow y(\infty) = 0$ only if V_y is nonsingular.

if : Suppose $G_{zw} \in \mathcal{RH}_\infty$. Then, it has already been shown that (12) holds. $y(\infty) = 0$ follows immediately since V_y is nonsingular. □

Remark 1 *As V_y is nonsingular, it follows that the number of rows of T_W must be $n_1 = m - n_z$.*

Remark 2 *In the controller design, a good starting point to tune T_W is to choose it satisfying the relation $T_W \perp V_2$.*

4 Multi area frequency control of power plants

The frequency control problem of multi area power systems is an important age-old problem in power industry[1, 3]. The main issue of this problem is to attenuate the frequency deviations due to the change of load (modeled as step disturbance). And the control system must be robust to parameter uncertainties as well.

The plant under consideration is a two area electric plant, with two generators, two loads and a tie-line connecting them. Its block diagram is shown in Fig. 2. See [1] for a detailed description of this model.

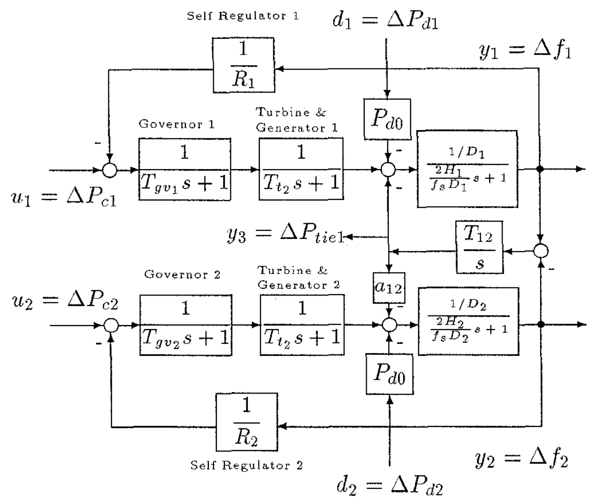

Figure 2: Block diagram of the plant

The plant has 3 measured outputs y_1, y_2, y_3 (frequency deviations in area 1, 2 and tie-line power).

For all analysis in this paper, we use the typical numerical data shown below, which is the same as in [1].

$$\begin{aligned} H_1 = H_2 &= 5 \text{ sec} \\ D_1 = D_2 &= 8.33 \times 10^{-3} \text{ MW/Hz} \\ T_{t1} = T_{t2} &= 0.3 \text{ sec} \\ T_{gv1} = T_{gv2} &= 0.08 \text{ sec} \\ R_1 = R_2 &= 2.4 \text{ Hz/pu MW} \\ T_{12} &= 0.545 \text{ pu MW} \\ a_{12} &= -1.1 \\ f_s &= 50\text{Hz} \\ P_{d0} &= 0.01 \text{ pu MW} \end{aligned} \quad (13)$$

By using a state space realization of this plant, it can be easily verified that this plant has a zero at $s = 0$[7] and the condition for disturbance rejection (see Theorem 1(ii)) is satisfied.

Performance Specification:

1. The static frequency deviation due to a step-load disturbance must be zero, $\Delta f_i(\infty) = 0 \quad (i = 1,\ 2)$.
2. The static change in the tie-line power due to a step load disturbance must be zero, $\Delta P_{\text{tie}}(\infty) = 0$.
3. The transient frequency deviation should not exceed ± 0.02 Hz for an unit step load disturbance, $\max_t(|\Delta f_i(t)|) < 0.02 \quad (i = 1,\ 2)$.

4.1 Generalized Plant

According to Theorem 2, we select T_W as

$$T_W = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{14}$$

The corresponding block diagram of the generalized plant is depicted in Fig. 3, in which the control input u is also penalized.

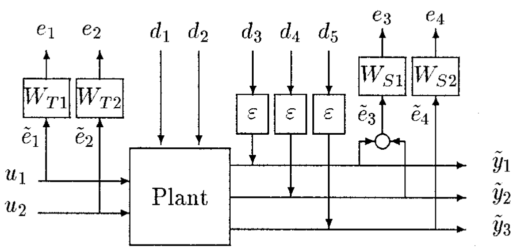

Figure 3: Block diagram of generalized plant G_Δ

4.2 Treatment of parameter uncertainty

By analyzing the Bode plots of the plant for both the nominal model and for the model with parameter uncertainty, it can be checked that the parameters that affect the frequency characteristics of the plant strongly are H_1, H_2 and T_{12}. These parameters arise from physical plant model and are subject to uncertainty, so it is desirable that the controller keep stability and performance robustly with respect to these parameters.

The parameter variations are modeled as follow

$$\begin{aligned} H_1 &= H_{10} + \delta_1 \Delta H_1 \\ H_2 &= H_{20} + \delta_2 \Delta H_2 \\ T_{12} &= T_{120} + \delta_3 \Delta T_{12} \end{aligned} \tag{15}$$

where H_{10}, H_{20} and T_{120} are the nominal values given in (13) and $-1 \leq \delta_i \leq 1$. We consider a variation of 20% in these parameters, that is,

$$(\Delta H_1, \Delta H_2, \Delta T_{12}) = 0.2 \times (H_{10}, H_{20}, T_{120}). \tag{16}$$

Now, by using the formulae given in [2], we can pull out δ_i and convert the generalized plant G_Δ of Fig. 3 with uncertain parameters to the LFT form $G_\Delta = \mathcal{F}_u(G_P, \Delta)$, with known system G_P and uncertainty $\Delta = diag(\delta_1, \delta_2, \delta_3)$, as shown in Fig. 4. The robust disturbance attenuation is achieved if the $\mathcal{H}_\infty$ norm of the closed loop transfer matrix $(w, d) \rightarrow (z, e)$ is less than 1.

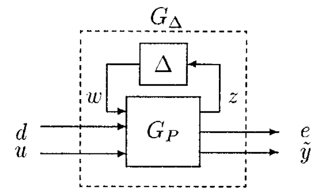

Figure 4: G_Δ and G_P

4.3 Simulation

The weights are chosen, via trial and error, as

$$\begin{aligned} &W_{S1} = W_{S2} = (0.5s + 1)/s \\ &W_{T1} = W_{T2} = 1 \\ &\varepsilon = 1 \times 10^{-6}. \end{aligned} \tag{17}$$

The $\mathcal{H}_\infty$ controller derived is of order 9, which is reduced to order 7 in simulation.

The plant is perturbed by 3 scalar parameters δ_i. So $P = P(\delta_1, \delta_2, \delta_3)$. The nominal model corresponds to $P(0,0,0)$. The controller robustness should be tested for the Δ that produces the worst performance. Testing in simulation, by varying δ_i, it is found that the combination $\Delta_1 = diag(1, -1, -1)$ is the case where the closed loop performance with respect to step disturbance d_1 decreases mostly. We will call $P(\Delta_1)$ the "worst case plant".

In Fig. 5 simulations are shown with the $\mathcal{H}_\infty$ controller applied. The left side is the result with respect to the nominal plant $P(0,0,0)$. The right side is the result for the worst case plant $P(\Delta_1)$. The upper part is the output y and the lower part is the input u.

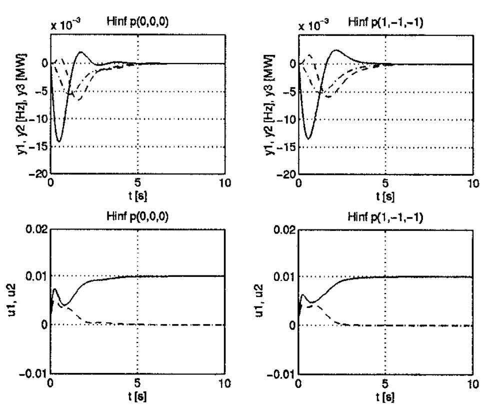

Figure 5: Unit step d_1 response for $\mathcal{H}_\infty$ controller. y_1 (solid) and y_2 (dashed) in Hz, y_3 (dash-dot) in MW, u_1 (solid) and u_2 (dashed) in control units, time in seconds

In Fig. 6 the LQG state feedback controller of [1] is applied to the nominal plant $P(0,0,0)$ (left) and to the worst case plant $P(\Delta_1)$ (right).

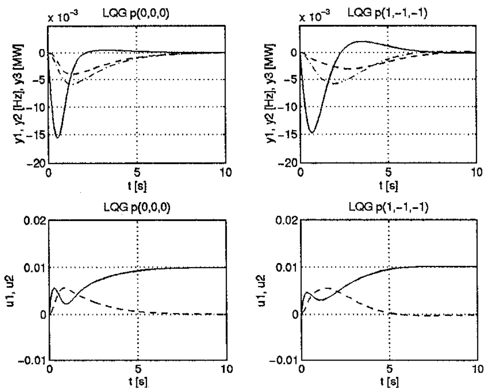

Figure 6: Unit step d_1 response for LQG state feedback controller. y_1 (solid) and y_2 (dashed) in Hz, y_3 (dash-dot) in MW, u_1 (solid) and u_2 (dashed) in control units, time in seconds

To have a numerical base for comparison we use the ITAE performance index. ITAE is defined below

$$ITAE = \int_{t=0}^{\text{final time}} t \cdot (|y_1| + |y_2| + |y_3|)dt. \quad (18)$$

The performance of the $\mathcal{H}_\infty$ controller for nominal plant model ($ITAE_0$) has the best performance, so we calculate the relative performance decrease for the remaining simulated responses as below.

$$\Delta ITAE = \frac{ITAE - ITAE_0}{ITAE_0} \quad (19)$$

	$P(0,0,0)$	$P(\Delta_1)$
$\mathcal{H}_\infty$ ITAE ($\Delta ITAE$)	1.96 (0)	2.31 (18%)
LQG ITAE ($\Delta ITAE$)	3.94 (101%)	5.54 (182%)

As is shown in both the time response and the ITAE table, the $\mathcal{H}_\infty$ controller performs better than LQG for nominal model, but the LQG has also acceptable response in this case. For the perturbed plant, for both controllers there is a deterioration of performance. The performance of $\mathcal{H}_\infty$ controller decreases much less than that of LQG controller. The performance of $\mathcal{H}_\infty$ controller for perturbed plant is even better than LQG for nominal plant.

5 Conclusion

We have shown that the zeros at $s = 0$ impose a constraint on the steady state output of plant and that reference tracking/disturbance rejection is possible only if certain structural condition is satisfied. Due to the existence of this zero, the output unstable weight used in the servo synthesis must also be subject to a structural constraint. This constraint has also been exposed.

If the constraint on the output weighting function is satisfied, one can use Ustably Weighted $\mathcal{H}_\infty$ Control theory[6, 5] to design a controller achieving asymptotic tracking/disturbance rejection.

This method has been applied to the $\mathcal{H}_\infty$ design of a frequency controller for a two-area power plant to reject step load disturbances asymptotically.

It has been shown in simulation that $\mathcal{H}_\infty$ controller performs much better than LQG state feedback controller of [1].

References

[1] C.E.Fosha and O.I.Elgered : The Megawatt-Frequency Control Problem: A New Approach Via Optimal Control Theory ; IEEE Trans. of Power Apparatus and Systems, Vol. 89, No. 4, pp. 563–577, (April 1970)

[2] M. Hirata, K. Z. Liu and Tsutomu Mita : Active Vibration of a 2-Mass Spring System using μ Synthesis with Descriptor Form Representation ; Control Engineering Practice, Vol. 4, No. 4, pp. 545–552, (1996)

[3] U. Kiffmeier and Mr. U. Keuchel : Comparison of Classical PID and Modern $\mathcal{H}_\infty$ Control Concepts for frequency and voltage control of power plants ; IFAC Control of Power Plants and Power Systems, Munich, Germany, pp. 97–102 (1992)

[4] K. Z. Liu, T. Mita: A unified stability analysis for regulator and servomechanism problems, Proceedings of the 33rd CDC, pp. 4198–4203 (1994).

[5] K. Z. Liu, M. Hirata and T. Sato: All solutions to the $\mathcal{H}_\infty$ control synthesis problem with unstable weights, Proceedings of the 36th CDC, pp. 4641–4646 (1997).

[6] T. Mita, K. Kuriyama, K. Z. Liu: $\mathcal{H}_\infty$ Control with unstable weighting functions - a design method of $\mathcal{H}_\infty$ robust servo systems, Proceedings of 32nd CDC, pp. 650–655 (1993).

[7] S. B. Villas-Boas, T. Mita, K. Z. Liu: Extended $\mathcal{H}_\infty$ Controller Applied to Frequency Control of High-Power Generation System, Proceedings of 35th CDC, pp. 761–766 (1996).